Mathematical Analysis 1 (Calculus 1), Shanghai University

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Website
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Text book

- Tom M. Apostol, Calculus Volume 1, One-Variable Calculus, with an Introduction to Linear Algebra, Second edition, Volume 1.

- James Stewart, Calculus Early Transcendentals, Brooks Cole

- R. A. Adams, Christopher Essex, Calculus - A Complete Course, Pearson Canada

Exercises

- Small exercises/examples/incomplete proofs will be scattered around during classes. Solve them yourself.

- Additional exercise sheets will be shared also

Calendar of the course

1st week

- Class 01, Monday September 6,   8.00-8.45am; 8.55-9.40am
- Class 02, Wednesday September 8, 8.00-8.45am; 8.55-9.40am
- Class 03, Friday September 10,   8.00-8.45am; 8.55-9.40am

2nd week

- Class 04, Monday September 13,   8.00-8.45am; 8.55-9.40am
- Class 05, Wednesday September 15, 8.00-8.45am; 8.55-9.40am
- Class 06, Friday September 17,   8.00-8.45am; 8.55-9.40am
- Class 07, Saturday September 18,   8.00-8.45am; 8.55-9.40am

3rd week

- ~~Class 07, Monday September 20,   8.00-8.45am; 8.55-9.40am~~ CANCELED
- Class 08, Wednesday September 22, 8.00-8.45am; 8.55-9.40am
- Class 09, Friday September 24,   8.00-8.45am; 8.55-9.40am
- Class 10, Sunday September 26,   8.00-8.45am; 8.55-9.40am

4th week

- Class 11, Monday September 27,   8.00-8.45am; 8.55-9.40am
- Class 12, Wednesday September 29, 8.00-8.45am; 8.55-9.40am
- ~~Class 13, Friday October 1,   8.00-8.45am; 8.55-9.40am~~ CANCELED

5th week

- ~~Class 14, Monday October 4,   8.00-8.45am; 8.55-9.40am~~ CANCELED
- ~~Class 15, Wednesday October 6, 8.00-8.45am; 8.55-9.40am~~ CANCELED
- Class 13, Friday October 8,   8.00-8.45am; 8.55-9.40am

Lectures 1-3, Introduction, basic Set Theory

Introduction
Parts 2,3,4, Apostol
Sections Preliminaries, Chapter 3, Adams
Appendix, Stewart

- Presentation of the course

- Introduction to Set Theory

- Set operations

- Order relations

- Sets of numbers, $\mathbb{N}, \mathbb{Z}, \mathbb{Q}$

- Axiomatic construction of $\mathbb{R}$

- Proof $\sqrt{2}$ is not a rational number

- Density of $\mathbb{Q}$ in $\mathbb{R}$

- Maximum, upper bound, supremum

- Absolute value

- Inequalities, triangle inequality

Lectures 4-6, Functions

Paragraph 1.2, page 50-57; Chapter 6 Apostol
Sections Preliminaries, Adams
Chapter 1, Stewart

- Functions, domain, codomain

- Even, odd functions. Examples, constant functions, linear functions, polynomial functions

- One-to-one, inverse function

- Elementary functions: trigonometric functions, hyperbolic functions, exponential, logarithm

Lectures 7-9, Complex numbers

Chapter 9, Apostol
Appendix, Adams
Appendix A.57, Stewart

- Algebraic, trigonometric, exponential notation

- Basic algebra of the complex numbers

- De Moivre's formulas

- Fundamental Theorem of Algebra

- Complex equations

- Induction principle

Lecture 10-11, Combinatorics

Paragraph 9.8, Adams

- Permutations with no repetitions

- Permutations with repetitions

- Combinations

- Binomial Theorem

- Pascal's triangle

Lectures 11-14, Sequences

Section 10.2 Apostol
Chaper 9, Adams
Chapter 11, Stewart

- Combinatorics

- Real sequences as functions $\mathbb{N} \to \mathbb{R}$

- Main properties of sequences (boundedness, monotonicity)

- Limit of a sequence

- Relevant examples and theorems for sequences

- Neper's number

- Cauchy sequence

- Bolzano-Weierstrass theorem

- Series

Continuity

Chapter 3, Apostol
Chapter 1, Adams
Chapter 2, Stewart

- Continuity

- Neighborhoods

- Limits

- Properties of limits

- Continuity of f at a point, continuous functions

- Main theorems for continuous functions, Bolzano's theorem, Intermediate value theorem, Weierstrass Theorem

- Uniform continuity

- Heine-Cantor theorem

- Lipschitz functions

- Criteria for computing limits, infinitesimal and infinite, Landau's symbols

Differential calculus

Chapter 4, Apostol
Chapter 2, Adams
Chapter 2,3,4, Stewart

- Definition of derivative, interpretation, examples and counterexamples

- Calculation of relevant derivatives

- Algebra of derivatives

- Chain rule, derivative of inverse function

- Extremum points, stationary points

- Rolle's theorem

- Mean-value theorems for derivatives

- Higher-order derivatives

- De l'Hopital theorem

- Second-derivative test for extrema

Polynomial approximations to functions

Chapter 7, Apostol
Chapter 9, Adams
Chapter 11, Stewart

- Taylor approximation

- MacLaurin expansion

- Study of a function, qualitative properties, approximations, convexity, monotone functions, limits, asymptotes,

Integration

Chapters 1,2,10, Apostol
Chapter 5,6,7 Adams
Chapter 5,6,7,8, Stewart

- Primitive of a function

- Riemann Integral

- Computation of areas

- Criterion of integrability for Riemann integral

- Main properties of Riemann Integrals

- Main theorems for the Riemann Integral

- Definite integrals, Fundamental theorem of calculus

- Indefinite integrals

- Integration formulas

- Techniques of integration