LECTURE 1, SEPTEMBER 6

THE MAIN CONTENT OF CALCULUS 1 IS 1-VARIABLE FUNCTIONS PROPERTIES (LIMITS, CONTINUITY, DERIVATIVES, INTEGRAL CALCULUS)

— BASIC SET THEORY —

SET = COLLECTION OF OBJECTS}

$A = \{$STATES OF U.S.A.$\}$

A

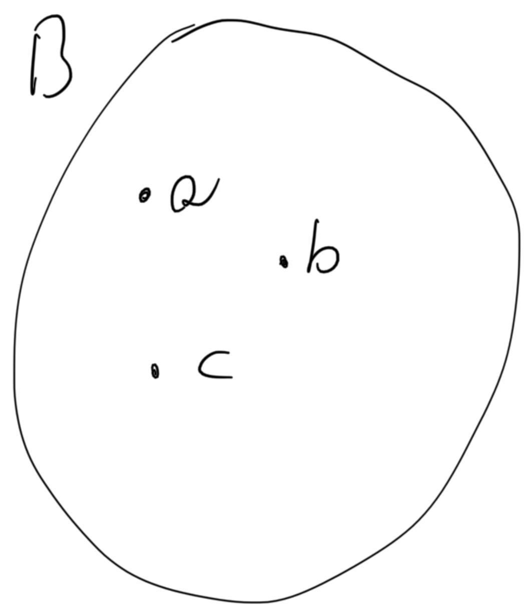
DENOTE SETS WITH UPPER CASE A, B, C

DENOTE ELEMENTS WITH LOWER CASE a, b, c

$a \in A$

↑ BELONGS

$a \in B,\ b \in B,\ c \in B$



$\mathcal{U}$ = "UNIVERSE"
= SET THAT CONTAINS ALL THE ELEMENTS.

$C = \{$NATURAL NUMBERS BIGGER THAN 1 AND SMALLER THAN 10$\}$

$C = \{2, 3, 4, 5, 6, 7, 8, 9\}$

$\mathcal{U} = \{$SET OF ALL NATURAL NUMBERS$\}$

$= \{1, 2, 3, 4 \ldots\}$

$2 \in C$ 2 BELONGS TO C

$1 \notin C$ 1 DOES NOT BELONG TO C

$C \subseteq \mathcal{U}$

C IS CONTAINED IN THE SET $\mathcal{U}$

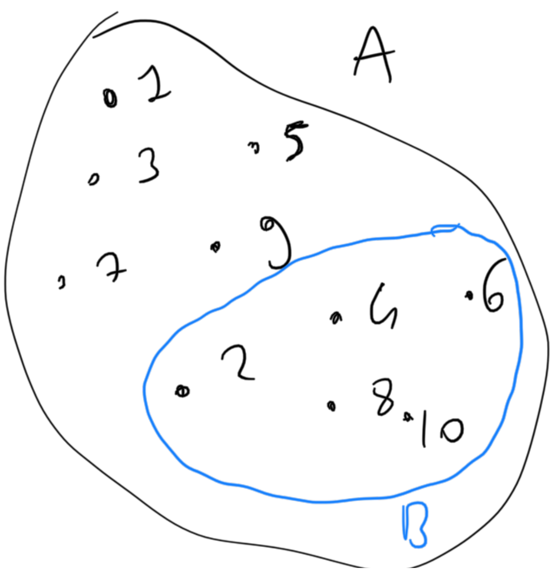
ALL THE ELEMENTS OF $\subset$ ARE ALSO ELEMENTS OF $\mathcal{U}$

$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$B = \{2, 4, 6, 8, 10\}$

$A = \{$NATURAL NUMBERS, BETWEEN 1 AND 10, INCLUDED$\}$

$B = \{$EVEN NUMBERS, BETWEEN 1 AND 10, INCLUDED$\}$



$B \subseteq A$

INCLUSION

$B \subset A$

STRICT INCLUSION

"$B \subset A$" STRICT INCLUSION →

THERE EXISTS AT LEAST AN ELEMENT OF A WHICH DOES NOT BELONG TO B

IN THIS EXAMPLE $1, 3, 5, 7, 9 \in A$

$\notin B$

→ $B \subset A$

STRICTLY CONTAINED IN A

$B \subseteq A$ OR

$A \supseteq B$

"CONTAINS"

$A \supset B$

STRICTLY CONTAINS B

REMARK

A, B $\quad A \subseteq B$ AND $B \subseteq A$

THEN WE HAVE $A = B$

WHICH MEANS A, B HAVE SAME ELEMENTS

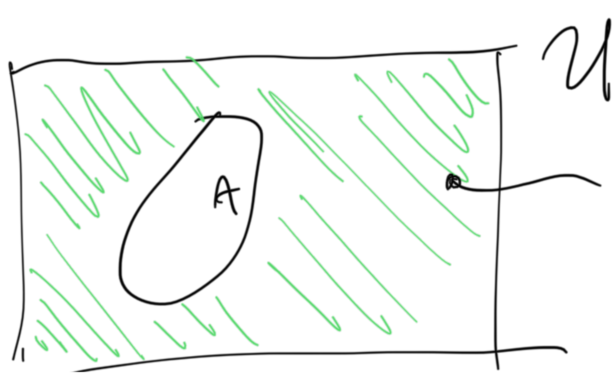
ONE WAY TO SHOW THAT TWO SETS ARE EQUAL IS TO SHOW DOUBLE INCLUSION

$\emptyset$ = SET WHICH DOES NOT CONTAIN ANY ELEMENT, EMPTY SET.

DEFINITIONS

1) COMPLEMENTARY OF A

$A^c = \{x \in \mathcal{U}$ AND DO NOT BELONG TO A



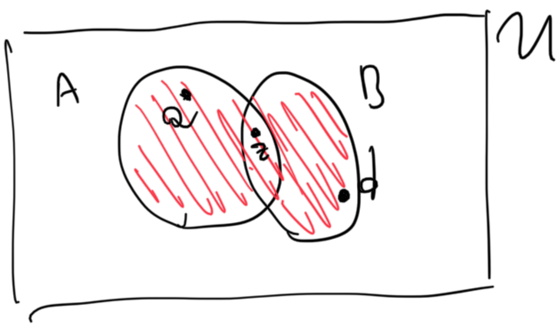
A^c IS THE SET THAT OCCUPIES THE GREEN AREA

2) UNION OF TWO SETS, A, B

$A \cup B := \{x \in \mathcal{U} : x$ BELONGS TO A OR x BELONGS TO $B\}$

$:= \{x \in \mathcal{U} : x \in A \vee x \in B\}$

OR



3) INTERSECTION OF TWO SETS A, B

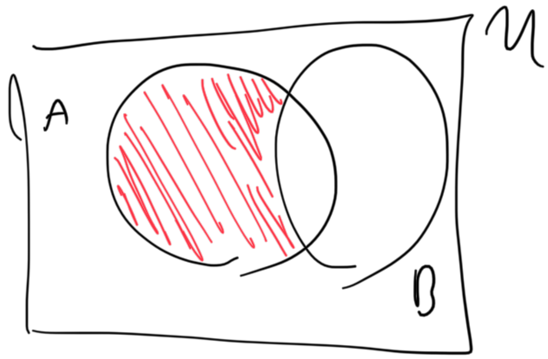
$A \cap B := \{x \in \mathcal{U} ; x$ BELONGS TO A AND x BELONGS TO $B\}$

$:= \{x \in \mathcal{U} ; x \in A \wedge x \in B\}$

AND

4) DIFFERENCE BETWEEN TWO SETS

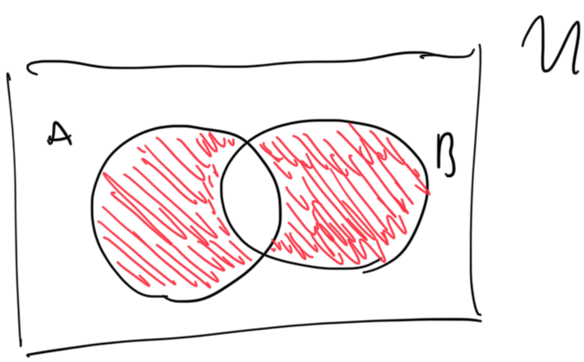
$$A \setminus B := \{x \in U \; ; \; x \in A \text{ AND } x \notin B\}$$



5) SYMMETRIC DIFFERENCE BETWEEN A, B

$$A \triangle B := \{x \in U : (x \in A \overset{\text{AND}}{\wedge} x \notin B) \overset{\text{OR}}{\vee} (x \in B \wedge x \notin A)\}$$

TRIANGLE



REMARK $A \triangle B = (A \cup B) \setminus (A \cap B)$

PROOF → TRY YOURSELF

CARTESIAN PRODUCT OF A, B

$$A \times B := \{(a,b) : a \in A, b \in B\}$$

$$A = \{1, 5, 10\} \qquad B = \{m, n\}$$

$$A \times B = \{(1,m), (1,n), (5,m), (5,n), (10,m), (10,$$

<u>REMARKS</u> UNION, INTERSECTION, SYMMETRIC DIFFERENCE ARE COMMUTATIVE

$$A \cup B = B \cup A$$

$$A \cap B = B \cap A$$

$$A \triangle B = B \triangle A$$

CARTESIAN PRODUCT IS NOT COMMUTATIVE

$A \times B \neq B \times A$

$B \times A = \{ (m, 1), (m, 5), (m, 10), (n, 1), (n, 5), (n, 10),$

$(1, m) \neq (m, 1)$

ELEMENTS OF $A \times B$, $B \times A$ ARE ORDERED PAIRS