Exercises, September 13, 2021

Upper and lower bounds

Exercise 1

Let

$$A = \{\frac{m}{n} + \frac{n}{m} : m, n \in \mathbb{N}\backslash\{0\}\}$$

and

$$B = \{(-1)^n \frac{n+1}{n}, n \in \mathbb{N}\backslash\{0\}\}.$$

Compute supremum, infimum, maximum and minimum of A and B as subsets of $\mathbb{Q}$.

Let

$$C = \{x^2 \leq 5 \text{ and } x < \pi, x \in \mathbb{R}\}.$$

Compute supremum, infimum, maximum and minimum of C as a subset of $\mathbb{R}$.

Solution. $\min A = 2$ and this coincides with the infimum, obviously. A is unbounded above so there is no supremum/maximum (equivalently, $\sup A = +\infty$).

$\min B = -2$ and this coincides with the infimum, obviously. $\max B = -2$ and this coincides with the supremum, obviously.

First of all, we need to identify the set C precisely. According to the definition we have $-\sqrt{5} \leq x \leq \sqrt{5}$ and $x < \pi$ which, together, gives:

$$C = \{x \in \mathbb{R} : -\sqrt{5} \leq x < \pi\}.$$

Then, $\min C = -\sqrt{5}$ and this coincides with the infimum, obviously. $\sup C = \pi$ while C has no maximum.